PROBABILISTIC FAILURE EVALUATION OF CYLINDRICAL PRESSURE VESSEL ON A THREE PARAMETER CRITERION USING MONTE CARLO SIMULATION

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Abstract Deterministic fracture analysis of cracked structures has been used in demonstrating the behaviour of structures under observed or postulated flaws. The key ingredients in a deterministic fracture mechanics analysis are the initial crack size, crack-driving force, applied stress and material properties. However, crack size, postulated accidental loading, mechanical and fracture parameters are often subjected to considerable scatter or uncertainty. Hence, the result of fracture mechanics analysis must be viewed with some scepticism. Conservative bounds on inputs are employed to account the uncertainty in the case of deterministic fracture analysis. However, it may lead to predict the overly conservative and unrealistic results. These uncertainties encourage us to adopt a statistical or probabilistic approach to the structural integrity. This paper presents the determination of failure probabilities of a cylindrical pressure vessel with an axial through crack, using a three parameter criterion. The scatter in the crack size, material properties and geometric parameters of the structure are considered. The probability of failure is studied for variation in crack size. Monte Carlo simulation is used to find the failure probability. The study is performed by constructing failure assessment diagram (FAD) using a three parameter criterion.

***Keywords**: Fracture Analysis, Cracked structures, Monte Carlo Simulation, Fracture strength*

I. INTRODUCTION

In recent years, probabilistic fracture mechanics (PFM) is becoming increasingly popular for realistic evaluation of fracture response and reliability of cracked structures. Using PFM, one can incorporate statistical uncertainties in engineering design and evaluation of a need, which has long been recognized. The theory of fracture mechanics provides a mechanistic relationship between the maximum permissible load acting on a structural component to the size and location of a crack (either real or postulated) in that component. The theory of probability determines how the uncertainties in crack size, loads, and material properties, if modelled accurately, affect the integrity of cracked structures. PFM, which blends these two theories, accounts for both mechanistic and statistical aspects of a fracture problem, and hence, provides a more rational way of describing the actual behaviour and reliability of structures than the traditional deterministic models.

II. NOMENCLATURE

a = crack depth

c = half the crack length

D_o, D_i = outer and inner diameter of cylindrical vessel

K_F, m, p = three-fracture parameters in (1)

K_{max} = stress intensity factor at failure

P_b = failure pressure of unflawed cylindrical vessel

P_{bf} = failure pressure of flawed cylindrical vessel

P_i = internal pressure

R = inner radius of cylinder

t = thickness of cylinder

W = specimen width

σ_f = failure stress

σ_u = nominal stress required to produce a fully plastic region on the net section

σ_{ult} = ultimate tensile strength

σ_{ys} = yield stress or 0.2% proof stress

III. FUNDAMENTALS OF PROBABILISTIC APPROACH

The PFM can be used to determine failure probabilities (P_f) of components by treating the scatter of applied loads, structural geometries and material properties adequately. The failure behaviour of component is described by limit state function (LSF), g(x), depending on basic random variables x = (x_1, x_2. . . X_n) denotes the several parameters. By definition, g(x) < 0 implies failure condition whereas no failure occurs for g(x) > 0 and g(x) = 0 defines the limit state. Then the failure probability is obtained by integrating the probability density function (PDF) of respective basic variables x_i over the region of g(x) < 0 [1]. In order to estimate the failure probability, two techniques are generally used: the one is reliability index technique such as first order reliability method (FORM) and second order reliability method (SORM), and the other is simulation technique [8]. In FORM, through a linearization of the LSF at design point, an approximate failure probability can be determined as following well-known expression

$$P_f = \Phi(-\beta) = 1 - \Phi(\beta) \quad (1)$$

Where Φ is the cumulative standard normal distribution function and β the reliability index that represents the minimum distance between the origin of the space of basic variables and the design point on failure surface.

In SORM, the failure surface is approximated by a quadratic hyper-surface associated with the curvature of non-linear limit state around the minimum distance point. A simple closed-form solution for the probability computation using second-order approximation is given as follows:

$$Pf \approx \Phi(-\beta) \prod_{i=1}^{n-1} (1 + \beta\kappa i)^{-1/2} \quad (2)$$

Where κi *is* the i^{th} main curvatures of the limit state and the value of $\Pi (1 + \beta\kappa i)^{-1/2}$ a specific term of SORM called as the multiplication factor, even though the definitions of Φ and β are same with those in FORM.

The Monte Carlo simulation (MCS) method, also, can be used to estimate the failure probability. It generates sets of random variables according to the given probabilistic distributions of the basic variables and puts them into the LSF. Thereby, the failure probability can be determined by Eq. (3)

$$P_f = \frac{N_{failure}}{N_{target}} \quad (3)$$

Where N failure is the number of simulation cycles when the failure occurred and N target is the total number of simulation cycles.

IV. FAILURE ASSESSMENT DIAGRAM

The significant parameters affecting the size of a critical crack in a structure are the applied stress levels, the fracture toughness of the material, the location of the crack and its orientation. Because the stress intensity factor, K is a function of load, geometry and crack size, it will be more useful to have a relationship between the stress intensity Factor at failure (K_{max}) and the failure stress (σ_f). From the fracture data of cracked specimens for the estimation/ prediction of the fracture strength to any cracked configuration. The relationship between K_{max} and σ_f can be of the form [3],[4]

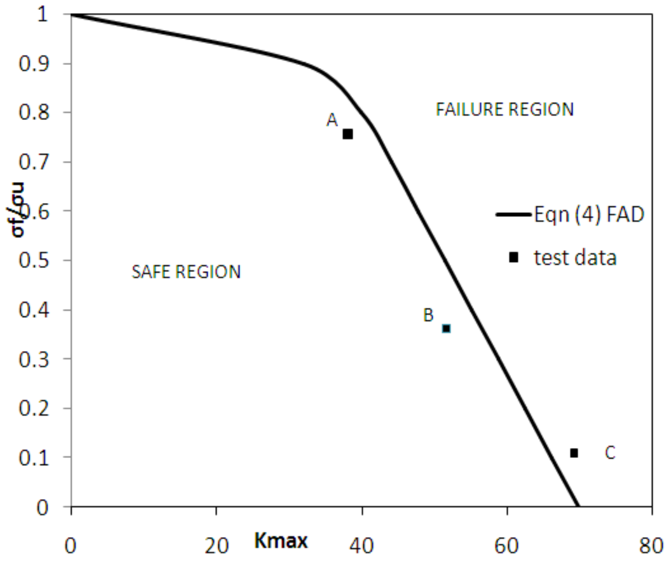

Figure. 2 Failure Assessment Diagram based on three parameter criterion

$$K_{max} = K_F \{1 - m\,(\sigma_f/\sigma_u) - (1 - m)\,(\sigma_f/\sigma_u)^{p}\} \quad (4)$$

Here, σ_f is the failure stress normal to the direction of a crack in a body and σ_u is the nominal stress required to produce a fully plastic region (or hinge) on the net section. For cylindrical pressure vessels, σ_u is the hoop stress at the burst pressure level of the unflawed thin cylindrical shell. K_F, m and p are the three fracture parameters to be determined from the fracture data. Fig.1 shows a cylindrical vessel containing an axial surface crack. Stress intensity factor expressions for these cracked configurations are available; [5-6] based on finite element solutions. Using the value of failure stress (σ_f) in the stress intensity factor expression, the stress intensity factor at failure (K_{max}) for the cracked configuration can be obtained. By substituting the stress intensity factor relation in Eq.(4) fracture strength expression for the cracked configurations in Fig. 1, is obtained expressions for K_{max} and σ_f are

$$K_{max} = \sigma_f\,(\pi a)^{1/2} M/\varphi \quad (5)$$

Where M and φ are correction factor relevant to the geometry chosen [2].

Fig. 1 shows the failure assessment diagram of a cylindrical pressure vessel with diameter of 142.2mm and thickness of 1.52 mm made from AA2014-T6 aluminium alloy. The x-axis represents the parameter K_{max}, and the y-axis represents the parameter σ_f/σ_u. The FAD is bounded by these two axes and the curve is called Failure Assessment Line (FAL).In this method, the analysis involves the assessment of given conditions and representing them by a single point, called the 'assessed point', on FAD and examining, where the assessed point lies in FAD. If a point falls inside the envelope, then it will indicate that the crack will not grow.

Table I. Failure Estimates of cylindrical pressure vessels

Crack size (2c) mm	Failure Pressure(P_{bf}) MPa
2.64 (A)	12.15
6.35	9.37
12.70(B)	5.85
19.05	4.73
25.40	3.10
31.75	2.93
44.45	1.95
50.80(C)	1.76

V. EVALUATION OF FRACTURE PARAMETERS

It is a well-known fact that the tensile strength, σ_f of a specimen decreases with increasing crack size. If $\sigma_f < \sigma_{ys}$, then there exists a linear relationship between σ_f and K_{max}. For small sizes of cracks where $\sigma_{ys} < \sigma_f < \sigma_u$, the relationship between σ_f and K_{max} is expected to be nonlinear. The idea of expressing K_{max} as a function of σ_f in a single expression (4) is mainly for estimation of the failure strength of a cracked body, whether it contains through thickness or part-through cracks, which are small or large in size. In the absence of cracks, $\sigma_f \rightarrow \sigma_u$ and $K_{max} \rightarrow 0$ (4) accounts for this limiting condition. The exponential form of the third term in (4) represents the nonlinear variation of K_{max} with σ_f, when $\sigma_f > \sigma_{ys}$. An empirical relation for the third fracture parameter *p*, in terms of the second fracture parameter *m*, has been derived [3], [4] as

$$p = 1/\ln \{1/2\,(1+\zeta)\ln [1/(1-m)\,\{1-1/2\sqrt{2}\,(1+\zeta)\,(1/\zeta + (\sqrt{2}-1)\,m)\}] \qquad (6)$$

$$\text{Where } \zeta = 43 + \sqrt{9} - 8m. \qquad (7)$$

With this empirical relation (6) and knowing σu, one needs to evaluate only K_F and *m* in (4) utilizing the fracture strength values from two cracked configurations. To account for the scatter in the experimental data, test results from a large number of cracked configurations should be fitted in (4) for obtaining the material parameters. The material parameter *m*, in general, is greater than zero and less than unity. If *m* is found to be less than zero due to a large scatter in the fracture data, then it has to be set to zero and the average of K_{max} from the fracture data yields the parameter K_F, and the third parameter *p*, from (6) gives a value close to 12.When *m* is close to unity, the third term in (4) becomes insignificant. Whenever *m* is found to be greater than unity, the parameter *m* has to be set to 1 by suitably modifying the parameter K_F with the fracture data. If the fracture-strength data are less than the yield strength of the material, then K_F and *m* in (4) can be obtained by fitting the fracture data in (4), neglecting the third term on the right hand side of (4). The third parameter, p is obtained using (6). If the fracture-strength values are higher than the yield strength, one has to obtain the K_F value in an iterative process by specifying m (between 0 and 1), evaluating p from (6) and fitting the fracture data in (4). This iterative process should continue until (4) satisfactorily correlates the fracture data with the obtained fracture parameters, K_F, m and p.

VI. PROBLEM DESCRIPTION

The objective of this paper is to study the effect of uncertainties with respect to crack size, material properties and geometric parameters of the structure on the existing FAD.

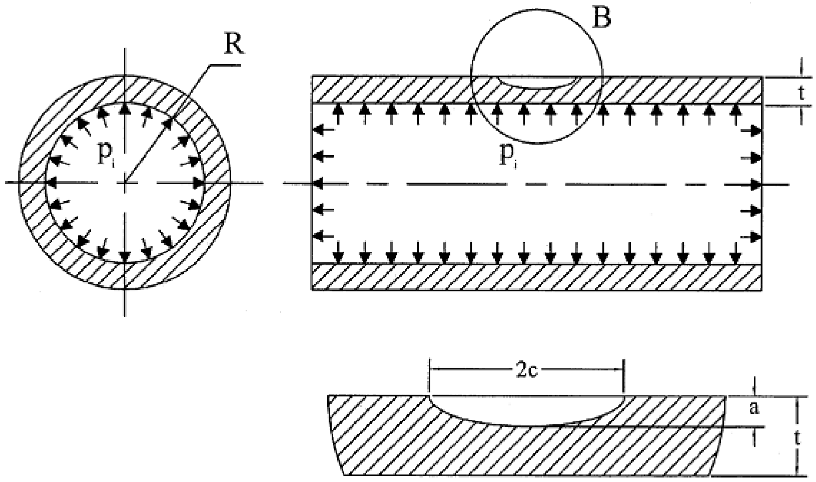

Figure .2 cylindrical shells with an axial surface crack under internal pressure

Table II. Statistical properties of random input for cylindrical pressure vessels

Variable	Mean	COV	Distribution
Crack length, 2c (mm)	2.64(A), 12.70(B), 50.80(C)	0.1	lognormal
Inner diameter, Di (mm)	142.2	0.02	lognormal
Thickness, t (mm)	1.52	0.02	lognormal
Yield strength, σ_y (MPa)	560	0.07	lognormal
Ultimate tensile strength, σ_u (MPa)	680	0.07	lognormal

A pressure vessel with an axial through crack of size 2C, subjected to internal pressure. The cracked pressure vessel is shown in Fig. 2. The failure mode considered is crack Growth Initiation using a three parameter criterion. The input data is obtained from a similar study carried out by [9] where the failure criteria used was *J*-Tearing. The input variables are assumed to follow log-normal distribution with mean and coefficient of variance given in Table. II Probability analysis is carried out using the Monte Carlo Simulation (MCS).

VII. STOCHASTIC ANALYSIS OF FRACTURE PARAMETERS

Probabilistic failure analysis is carried out based on a three parameter failure criterion using Monte–Carlo simulation technique. When the Monte–Carlo algorithm is used a very large number of simulations have to be taken in order to achieve high computational accuracy. In this 1000 Number of simulations have been taken with the computer code developed using C Language. The probabilistic failure analysis is carried out by the following steps.

1. using random number generator, generate a value for each of the random input parameters of the analysis.
2. Using the generated random vectors, K_{max} and σ_f/σ_u are evaluated
3. Plot all the K_{max} and σ_f/σ_u in FAD.

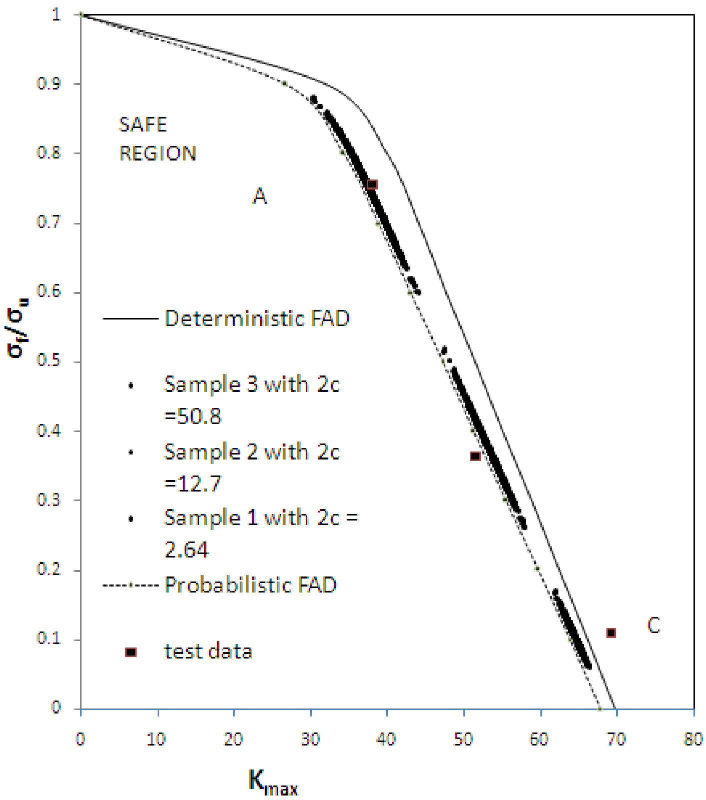

Figure. 3 Probabilistic Failure assessment Diagram

VIII. RESULTS AND DISCUSSION

Using a three parameter fracture criterion and probabilistic approach failure assessment points have been obtained by solving the equation through Newton Raphson method. These points have been plotted on the deterministic FAD already generated. Figure 3 shows the clusters of points lying close to the deterministic FAD. But, these points lie below the deterministic FAD, which shows the probabilistic assessment are safer. Using these points a corrected FAD has been generated and the same also is shown in Fig. 3.

The fracture parameters K_F, m and p for deterministic and corrected FAD as shown in Table. III

Table III. Fracture Parameters K_F, m and p

Case	K_F	m	p
Deterministic FAD	68.9	0.60	20.4
Corrected FAD	68	0.61	18.217

IX. CONCLUSION

Deterministic FAD does not give confidence since the parameters are always subject to scatter. A corrected FAD has been generated based on the probabilistic assessment points. Since the statistical scatter has been taken in to consideration, the corrected FAD can be used with confidence. By knowing the solution for stress intensity of any geometry, failure of that geometry can be assessed using the generalized FAD if material and thickness are the same.

REFERENCES

[1] Haldar, A., Mahadevan, S., 2000. "Probability, Reliability and Statistical Methods in Engineering Design. John Wiley and Sons Inc., USA.

[2] T.christopher, "Correlating cryogenic fracture strength using modified two – parameter method", in Engineering Fracture Mechanics72 (2005) 475-490.

[3] Rao, B. N. and Acharya, A. R. (1997) "Failure assessment on 34CrMo4 grade steel cylindrical pressure vessels with an axial crack" Int. J. Press. Vessels Pip. 72, 157–163.

[4] Potti, P. K. G., Rao, B. N. and Srivastava, V. K. (2000) "Residual strength of Aluminium–Lithium alloy center surface crack tension specimens at cryogenic temperatures" Cryogenics 40,789–795.

[5] Newman, J. C. Jr. and Raju, I. S. (1979) "Analysis of surface cracks under tension or bending loads", NASA-TP-1578.

[6] Newman, J. C. Jr. (1976) "Fracture analysis of surface and through cracks in cylindrical pressure vessels", NASA-TND-8325.

[7] Y.C. Lin, "Probabilistic fracture failure analysis of nuclear piping containing defects using R6 method", in Nuclear Engineering and Design 229 (2004) 237–246

[8] Sang-Min Lee, "Failure probability assessment of wall-thinned nuclear pipes using probabilistic fracture mechanics " in Nuclear Engineering and Design 236 (2006) 350–358

[9] S.Rahman, "Probabilistic fracture mechanics for non linear structures" in International journal of Pressure vessels and piping 78 (2001) 261 – 269

[10] Rohit Rastogi," Probabilistic fracture analysis of a straight pipe with through-wall circumferential crack using R6 method", in Nuclear Engineering and Design 212 (2002) 109–114

[11] Jian-Ping Zhao, "A new concept: probabilistic failure assessment diagram", in International journal of Pressure vessels and piping 71 (1997) 165-16X

[12] Jin Xing," A probabilistic fracture mechanics assessment method based on the R6 procedure", International journal of Pressure vessels and piping 73 (1997) 161-163

[13] Akramin. M.R.M , "Probabilistic analysis of linear elastic cracked structures", in J Zhejiang Univ Sci A 2007 8(11):1795-1799