STATISTICAL ANALYSIS OF FRACTURE STRENGTH OF COMPOSITE SPECIMEN USING WEIBULL DISTRIBUTION

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Abstract- In this study, the fracture strength of a composite specimen has been statistically analyzed by Weibull distribution. The fracture strength of the composite specimensis obtained by numerical simulations performed in ANSYS 11. The finite element analysis results from ANSYS are compared with the experimental results of tensile test on composite sheet [1]. In the probabilistic FEA model, the maximum stresses in fiber direction are obtained by using Monte Carlo Simulations. Finally the reliability of the composite behavior in terms of its fracture strength is presented to ensure the reliability of composites for suitable applications.

Keywords: *Weibull Distribution, Composite specimen, Fracture Strength, Finite Element Analysis.*

I. INTRODUCTION

In recent years conventional materials are replaced by composite materials due to their mechanical properties such as high specific strength, Elastic modulus, light weight and corrosion resistance. Aircraft and automobiles are examples of vehicles in which the application of fiber reinforced composite materials has been increasing.

However, there are problems with fabrication of these composites, which require high temperature methods like infiltration squeeze casting, hot-extrusion or hot- pressing. Whatever the method used for fabrication of composites, they are not isotropic and therefore have different mechanical properties in different directions. In addition to this, they present varying strengths due to their internal structure and to brittleness of the fibers and matrices, which means that there is no specific strength value to represent their mechanical behavior. This leads to the necessity of employing statistical analyses for their safe utilization in design and manufacturing.

TheWeibull distribution, which has recently been used for the determination of static and dynamic mechanical properties of ceramics and metal-matrix, ceramic- matrix, and polymer- matrix composites statistically. Weibull distribution is a practical method in the determination of 90% and 95% reliability values used in composite material mechanics. In this study, the fracture strength of the carbon/epoxy composite plates was determined probabilistically using ANSYS 11. The specimen modeled using PLANE42 element and dimensions are taken according to ASTM D3039 standard. The results are statistically analyzed by using Weibull distribution in order to predict the exact fracture strength of the composite specimen. The fracture strength results of the composite specimen are illustrated in the form of reliability function on the graph. The simulated results are compared with experimental results published in the literature [1].

II. EXPERIMENTAL DATA FROM LITERATURE

The composite specimens used in the experiments were prepared from carbon/epoxy sheets with (0^o) configuration, 0.89mm thickness and 295 g/m^2 weight. The mechanical properties present in Table 1. The tests were carried out according to ASTM D3039 standard (ASTM D3039, 1976) on an Instron 8516+ universal testing centre. A crosshead speed of 1.33 mm/min is used and room temperature conditions were present during the tests. The dimensions of the test specimens are showed in figure 1 and the fracture strength values obtained are given in table 2.

Table1. Mechanical properties of carbon-epoxy composite plate.

Mechanical properties	Values
E_1 (GPa)	40.74
E_2 (GPa)	39.60
G_{12} (GPa)	4.62
υ_{12}	0.25
Carbon (%)	28

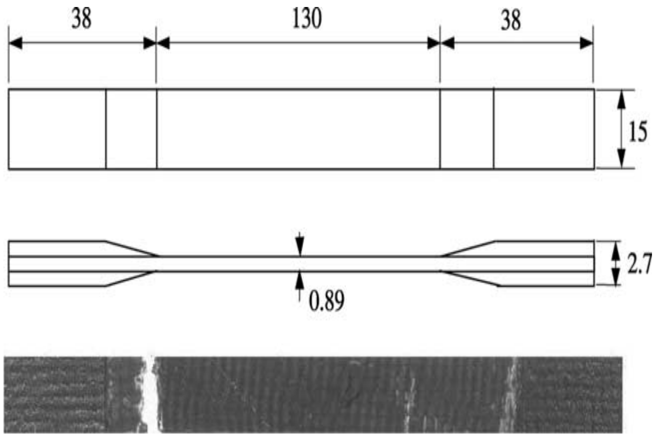

Figure 1. Test specimens and its dimensions in mm.

Table 2. Fracture strength values from tension tests.

Test No.	Fracture strength (MPa)
1	532.7
2	502.5
3	442.0
4	473.0
5	519.0
6	502.7
7	477.0
8	510.0
9	522.0
10	552.0
11	522.0
12	439.0
13	513.6
14	497.5
15	521.6
16	450.9
17	476.5
18	507.3
19	463.5

III. DETERMINISTIC FEA ANALYSIS

FEA Model

A 2-D solid model of the actual test specimen is built using ANSYS 11.0 software package using PLANE 42 element. PLANE 42 is used for 2-D modeling of solid structures. The element can be used either as a plane element (plane stress or plane strain) or as an ax symmetric element. The element is defined by four nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. The x-direction corresponds to the longitudinaldirection and loading direction. The y-direction corresponds to the lateral direction of the test specimen. In this study, one PLANE 42 finite element is used through the thickness of the specimen.

Model Restraints and Load Application

In this model at one end all degrees of freedom (Ux=Uy=0) is arrested and at another end the pressure 400 MPa is applied. The meshed model along with its restraints, load applications is shown in figure 2.

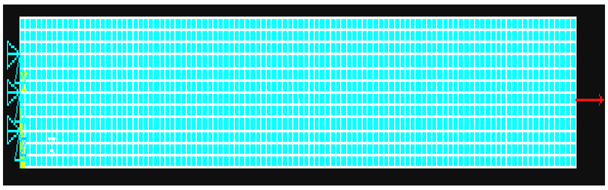

Figure 2: Finite element model of tension specimen.

Failure criteria

Tsai-wuapplied the failure theory to a lamina in plane stress. A lamina considered to be failed if

$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1(1)$

This failure theory is more general than the Tsai-Hill failure theory because it distinguished between the compressive and tensile strength of a lamina. The components $H_1, H_2, H_6, H_{11}, H_{22}$and H_{66} of the failure theory are found using the five strength parameters of a unidirectional lamina as follows,

$$H_1 = \frac{1}{(\sigma_1^T)_{ult}} - \frac{1}{(\sigma_1^C)_{ult}}$$

$$H_{11} = \frac{1}{(\sigma_1^T)_{ult}(\sigma_1^C)_{ult}}$$

$$H_2 = \frac{1}{(\sigma_2^T)_{ult}} - \frac{1}{(\sigma_2^C)_{ult}}$$

$$H_6 = 0$$

$$H_{66} = \frac{1}{(\tau_{12})_{ult}^2}$$

$$H_{12} = \frac{1}{2\sigma^2}[1 - (H_1 + H_2)\sigma_1 - (H_{11} + H_{22})\sigma^2]$$

$$H_{22} = \frac{1}{(\sigma_2^T)_{ult}(\sigma_2^C)_{ult}}$$

Where,

$(\sigma_1^T)_{ult}$= ultimate tensile strength (1)

$(\sigma_1^C)_{ult}$= ultimate compressive strength (1)

$(\sigma_2^T)_{ult}$= ultimate tensile strength (2)

$(\sigma_2^C)_{ult}$= ultimate compressive strength (2)

$(\tau_{12})_{ult}^2$= shear strength (1&2)

Probabilistic FEA Model using Monte Carlo Simulations

The Monte Carlo Simulation is a powerful tool for analyzing structures with property uncertainty. This method predicts the response of a structure subjected to variable design parameter of known or pre-described probability distributions. Using statistical sampling techniques a set of values of the basic random variables are generated according to their corresponding probability distributions. The generated basic random variables are used in the finite element model to compute the structure response.

The sampling procedure is repeated to obtain a multitude of simulated solutions. The random input variables consideredare modulus of elasticity (both directions), shear modulus and Poisson's ratio. The random output variables are maximum stress in fiber direction, shear stress in xy direction. The COV taken for variability is 20% for each variable. The simulated values are present in table 3.

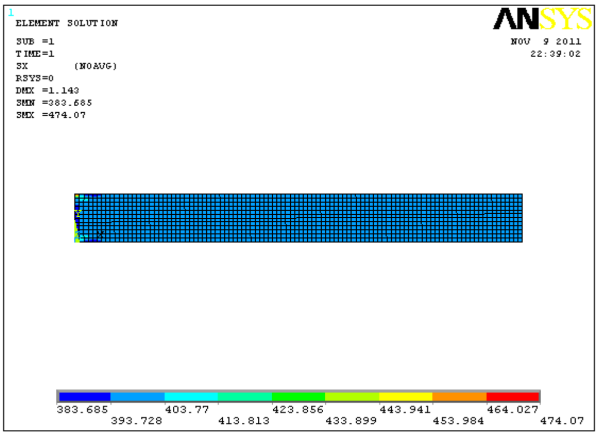

Figure 3.Fiber direction stress (maximum).

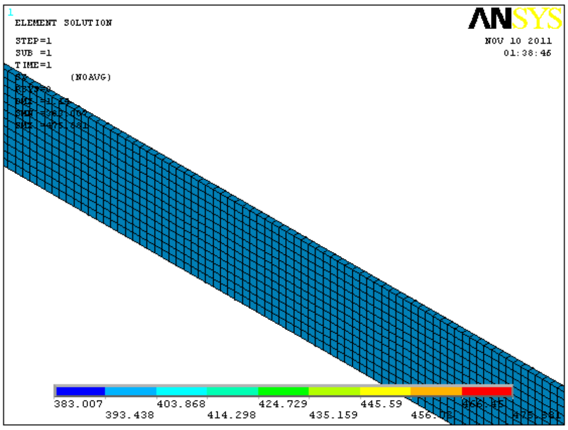

Figure 4. 3-D view of FEA model.

Table 3. Fracture strength values from numerical simulation.

Iteration No.	Fracture strength (MPa)
1	528.2
2	495.7
3	470.7
4	462.8
5	528.4
6	459.3
7	462.2
8	451.6
9	426.0
10	436.7
11	490.5
12	447.6
13	412.9
14	463.2
15	437.5
16	484.8
17	439.3
18	481.6
19	474.0

IV. STATISTICAL ANALYSIS OF COMPOSITE SPECIMEN

Weibull Distribution

Weibull distribution is being used to model extreme values such as failure times and fracture strength. Two popular forms of this distribution are two and three parameter Weibull distributions. The distribution function of the three parameter Weibull distribution is given as follows

$$F(x;b,c) = 1 - \exp\left(-\left(\frac{x-a}{b}\right)^{c}\right), a \geq 0,\ b \geq 0, c \geq 0 (2)$$

Where a, b and c are the location, scale and shape parameters, respectively. When a=0, the distribution function of the two-parameter Weibull distribution is obtained. The three-parameter Weibull distribution is suitable for situations in which an extreme value cannot take values less than 'a'. In this study, the two-parameter Weibull distribution, which can be used in fracture strength studies, will be considered. The distribution function can be written as

$$F(x;b,c) = 1 - \exp\left(-\left(\frac{x}{b}\right)^{c}\right), b \geq 0,\ c \geq 0\ (3)$$

The equality equation is given by

$$F(x;b,c) + R(x;b,c) = 1 (4)$$

The probability that the fracture strength is at least x is defined as

$$R(x;b,c) = \exp\left(-\left(\frac{x}{b}\right)^{c}\right), b \geq 0, c \geq 0\ (5)$$

The methods usually employed in the estimation of the Weibull parameters are method of linear regression, method of maximum likelihood, and method of moments (Mohammad A. Al-Fawzan, Methods for estimating the parameters of the Weibull distribution). Among these methods, method of linear regression is used.

Method of Linear Regression

This method is based on transforming equation into$1 - F(x;b,c) = \exp\left(-\left(\frac{x}{b}\right)^{c}\right)$ and taking the double logarithms of both sides. Hence, a linear regression model in the form $Y = m\ X + r$ is obtained.

$$\ln\left[ln\left(\frac{1}{1-F(x;b,c)}\right)\right] = cln(x) - cln(b)\ (6)$$

F(*x*; *b; c*) is an unknown in (4) and, therefore, it is estimated from observed values: order *n* observations from smallest to largest, and let x (*i*) denote the i^{th} smallest observation (*i*=1 corresponds to the smallest and *i*= *n* corresponds to the largest). Then a good estimator of $F(x_{(i)}; b; c)$ is the median rank of $x_{(i)}$:

$$\hat{F}\left(x_{(i)};b,c\right) = \frac{i-0.3}{(n+0.4)}\ (7)$$

When linear regression, based on least squares minimization, is applied to the paired values (X, Y)

$$\ln\left(x_{(i)}\right), \ln\left[ln\left(\frac{1}{1-\hat{F}\left(x_{(i)};b,c\right)}\right)\right]$$

By using this x, y values plot the regression line and find out the slope "c".

C < 1; Decreasing failure rate.
C = 0; Constant failure rate.
C > 1; Increasing failure rate.

The "b" value is computed by using$b = e^{\left(-\frac{Y}{c}\right)}$.
Where, Y is the line intersects the Y axis of regression plot.

V. RESULTS AND DISCUSSIONS

The results obtained from both experiments and numerical simulations in the present work are given in Tables 2 and 3. In order to compute '*b*' and '*c*' first, they are ordered from the smallest to the largest and (X, Y) values are computed. Then applying linear regression to these (X, Y) values, the linear regression model with the regression lines are in figures 5 and 6 are obtained. The slope value of the line for experimental data is 17.44 and for numerical data is 16.81, which is the values of the shape parameter. The *b* value is computed for experimental data as *b*= 510.76 using the point the line intersects the Y axis (-108.77), and for numerical data as *b*= 511.22 using the point the line intersects the Y axis (-104.83).

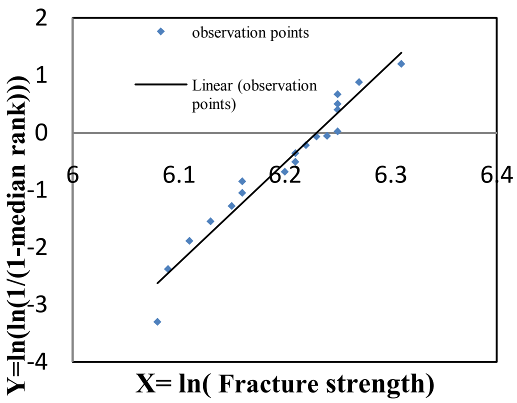

Figure 5.Regression line for experimental data.

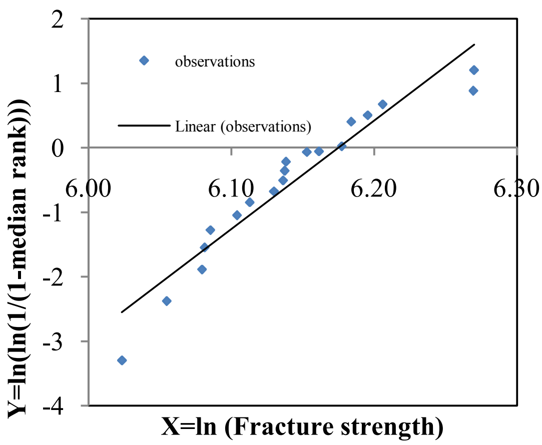

Figure 6. Regression line for numerical data.

Therefore, *c* indicates that the material tends to fracture with higher probability for every unit increase in applied tension. The scale parameter *b* measures the spread in the distribution of data. As a theoretical property $R(x; b, c)$= 0.368. Therefore,

$$R(510.76{:}510.76,17.44)= \exp\left(-\left(\frac{x}{b}\right)^{c}\right)=0.368,$$

That is 36.8% of the tested specimens have fracture strength of at least 510.76 MPa and 510.89 MPa.

The plots of $R(x; b, c)$ is shown in figures 7 and 8. The curve shows that fracture strength values roughly less than or equal to 400 MPa will provide high reliability. For a more certain assessment, consider 0.90 and 0.95 reliability levels. When these values are put as $R(x;b,c)$ in the reliability equation and the fracture strength values 448.92 and 430.79 are obtained respectively.

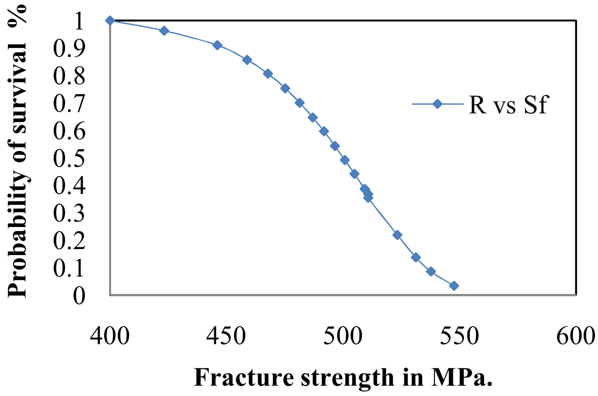

Figure 7. Weibull reliability distribution for experimental data.

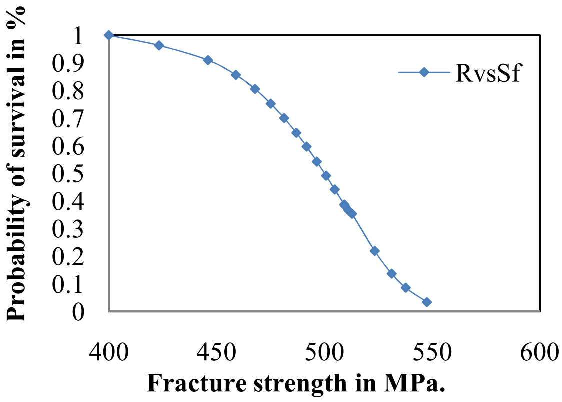

Figure 8.Weibull reliability distribution for numerical data.

VI. CONCLUSION

The fracture strength variation of carbon-epoxy composite in tension has been modeled using Weibull distribution. The experimental results were compared with numerical simulations obtained by probabilistic analysis using ANSYS. From the comparison, the numerical results are same. This study questions and then rejects the assumptions that the fracture strength of composite materials is taken as an average of the experimental results. In this respect, the Weibull distribution allows researchers to describe the fracture strength of a composite material in terms of a reliability function. It also provides composite material manufacturers with a tool that will enable them to present the necessary mechanical properties with certain confidence to end users.

VII. FUTURE WORK

The experimental work on glass-polyester composite specimen is going to be conducted and the results obtained will be compared with FEA results.

Table 4: Nomenclature

a	-	Location parameter
b	-	Scale parameter
c	-	Shape parameter
$F(x;b,c)$	-	Distribution function
$R(x;b,c)$	-	Reliability function
n	-	Observation number
$x_{(i)}$	-	i^{th} order statistic

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